5.4 Mathematical Derivation in the $7dU$ Framework

The 7-dimensional metric tensor $g_{\mu\nu}$, which includes contributions from the chance dimension, influences the conjugate operators $\hat{x}$ and $\hat{p}$. In the 7dU framework, the metric tensor is:[5]

$$g_{\mu\nu} =$$

$$\begin{bmatrix} g_{00} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi^2 \end{bmatrix},$$

where the ξ^2 term introduces a fluctuating geometry in the chance dimension. This fluctuation modifies the conjugate operators as follows:

$$\hat{x} \rightarrow \hat{x} + \xi\,\hat{\xi}_x, \quad \hat{p} \rightarrow \hat{p} + \xi\,\hat{\xi}_p,$$

where $\hat{\xi}_x$ and $\hat{\xi}_p$ are operators associated with the chance dimension

Substituting these modified operators into the standard commutator relation yields:

$$[\hat{x}, \hat{p}] = i\hbar + \xi\left([\hat{\xi}_x, \hat{\xi}_p]\right),$$

where $[\hat{\xi}_x, \hat{\xi}_p]$ encapsulates the influence of chance on the system's uncertainty. This term contributes directly to $g(\xi)$ in the generalized uncertainty relation.